# Errata etc. for *Algorithmic Learning in a Random World* (second edition, 2022)

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## 1 Errata

- **Page 302, 2nd line up from Lemma 9.14:** Remove "and any $\epsilon > 0$". (From the point of view of formal logic it might not be an error, but it is superfluous and misleading.) When we say "p-variable" at the end of this sentence, we, of course, mean a p-variable with respect to $P$.
- **Page 359, 3rd line up from bottom:** "Asarin [8]" should be "Asarin [9]".
- **Page 449, reference 8:** replace "$\delta$-random" by "$\Delta$-random".

## 2 Complements

- **Page 138, first line after the proof of Proposition 4.8:** We refer to "[385, proof of Theorem 1]". That proof is unnecessarily complicated; namely, Lemma 1 in [385] (in the book's bibliography) is easily obtained by the method of coupling [Dubhashi and Panconesi, 2009, Sect. 7.4].
- **Page 358, Sect. 11.6:** Here we should have mentioned that generalized conformal prediction was introduced in Vovk [2003].
- **Page 405, line 2:** We mention that Fisher regarded his verification protocol as somewhat unnatural but do not mention that the protocol given in [405, Sect. 9 of the arXiv report] is completely natural (since it involves testing predictions based on all the available data).
- **Page 473:** We should have included

  Probability integral transformation, 307, 327

  in the index.

# 3 Comments

## 3.1 Lemma 9.6 on p. 269

Lemma 9.6 is a special case of a result on the optimality of the likelihood ratio. Let $P$ and $Q$ be probability measures on the same sample space, playing the role of the null and alternative hypotheses, respectively. The *e-power* of an e-variable $E$ w.r.t. to $P$ is then defined as $\int \log E \, \mathrm{d}Q$.

**Lemma A.** *For given null and alternative hypotheses $P$ and $Q$, respectively, such that $Q \ll P$, the largest e-power is attained by the likelihood ratio $\mathrm{d}Q/\mathrm{d}P$: for any e-variable $E$,*

$$\int \log E \, \mathrm{d}Q \leq \int \log \frac{\mathrm{d}Q}{\mathrm{d}P} \, \mathrm{d}Q.$$

*And if $Q \ll P$ is violated, the largest e-power is $\infty$.*

The likelihood ratio $\mathrm{d}Q/\mathrm{d}P$ in Lemma A is understood to be the Radon–Nikodym derivative of $Q$ w.r.t. to $P$. For a simple proof, see, e.g., [Shafer, 2021, Sect. 2.2.1] or Vovk and Wang [2022].

Lemma 9.6 in the book is a special case of Lemma A in which $P$ is the uniform probability measure on $[0, 1]$ and $\rho = \mathrm{d}Q/\mathrm{d}P$.

## 3.2 Section 9.3 on pp. 294–298

The behaviour of the test martingales and related processes in this section appears anomalous; e.g., the mean in the majority of rows in Table 9.1 lies outside the interquartile range. In particular, the final values of our test martingales have heavy-tailed distributions. The reason is that we consider multiplicative processes, in the terminology of [Nair et al., 2022, Chap. 6].

# References


Devdatt P. Dubhashi and Alessandro Panconesi. *Concentration of Measure for the Analysis of Randomized Algorithms*. Cambridge University Press, Cambridge, 2009.

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